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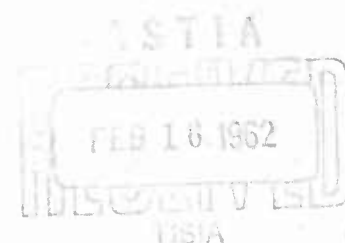
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TR-907

# HIDDEN TRANSMITTER LOCATION BY PULSE ARRIVAL MEASUREMENTS

Walter J. Brinks

22 March 1961



DIAMOND ORDNANCE FUZE LABORATORIES  
ORDNANCE CORPS • DEPARTMENT OF THE ARMY

WASHINGTON 25, D. C.

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**DIAMOND ORDNANCE FUZE LABORATORIES**  
**WASHINGTON 25, D. C.**

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**FOR THE COMMANDER:**  
**Approved by**

  
B. M. Horton  
Chief, Laboratory 200



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## ABSTRACT

A method is presented for determining the location of an uncooperative radar transmitter by timing the pulses of the transmitter after they have been reflected or transduced by a moving target of known location. A procedure is discussed for assessing the errors involved.

### 1. THE PROBLEM

The following problem has been proposed in the radiolocation field: Given a radar transmitter in a foreign location tracking a missile, usually a missile of foreign origin, assume that at a receiving point in a friendly location, it is possible to detect the radar pulses reflected from, or transponded by, the missile but it is not possible to hear the transmitter directly. The flight path of the missile is known at the receiver location, usually through friendly optical or radar tracking. The problem is to locate the radar transmitter.

Several assumptions must be made in order to solve this problem: The pulse repetition period  $T$  of the radar is taken as constant, although its value need not be known. If the pulse is transponded at the missile, the time delay of the transponder is also taken to be constant.

The location of the transmitter will be deduced from the missile position information together with data giving the time of arrival of the reflected or transponded pulses at the receiver.

### 2. THEORETICAL DISCUSSION AND SOLUTION

Consider the times of arrival  $\tau_n$  of the received pulses. The receiver time at which the  $n$ th pulse left the transmitter (fig. 1) is:

$$\tau_n - \left( \frac{H_n}{c} + \delta \right)$$

where  $H_n$  = total transmitter-to-missile-to-receiver distance

$c$  = velocity of light

$\delta$  = time delay of transponder

( $\delta = 0$  for a reflected signal)

Naturally, the missile position used in calculating  $H_n$  is that position the missile occupies when the  $n$ th pulse is operated upon.

Consider another pulse, the  $j$ th pulse; it is clear that it left the transmitter a period of time  $(n-j)T$  before the  $n$ th pulse, provided  $n \geq j$ . Therefore

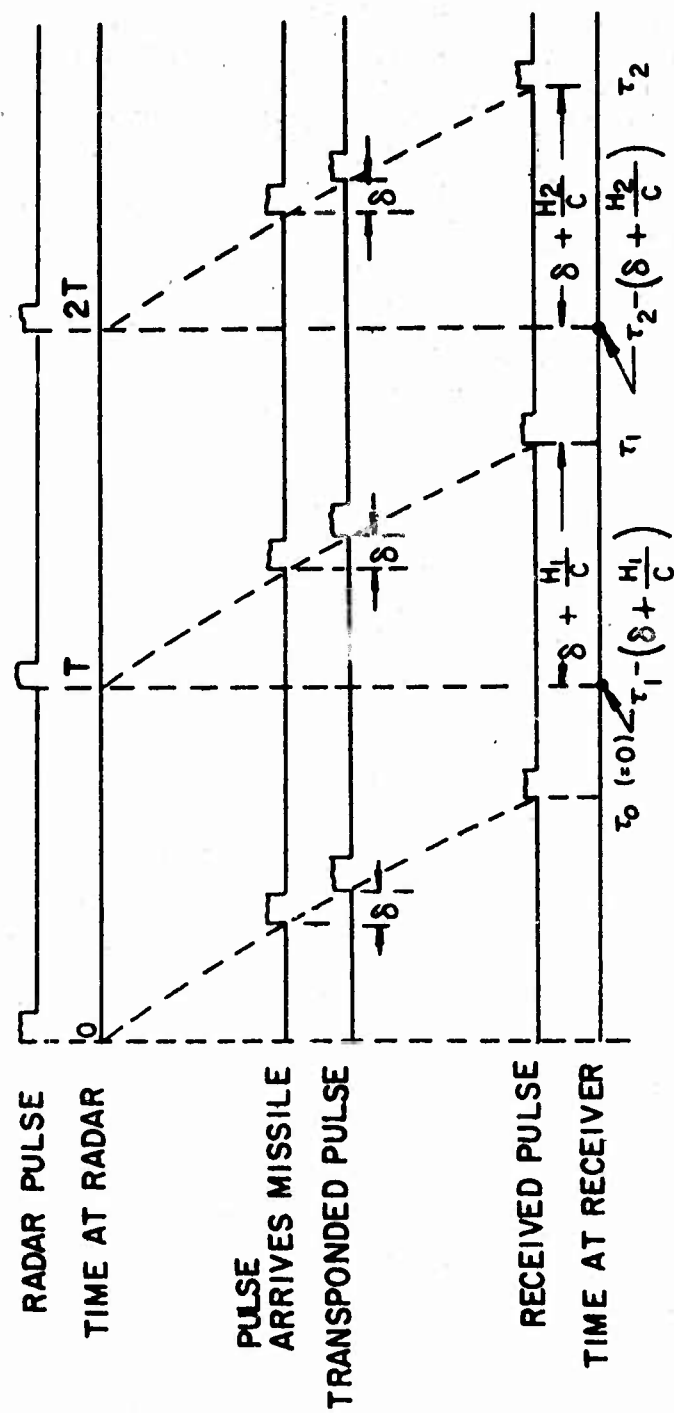


Figure 1. Time disposition of transmitted pulses.



$$\left[ \tau_n - \left( \frac{H_n}{c} + \delta \right) \right] - \left[ \tau_j - \left( \frac{H_j}{c} + \delta \right) \right] = (n-j) T$$

Now take two other pulses,  $m$  and  $i$ ,  $m > i$ , such that  $n-j = m-i$ . It follows at once that

$$\left\{ \left[ \tau_n - \left( \frac{H_n}{c} + \delta \right) \right] - \left[ \tau_j - \left( \frac{H_j}{c} + \delta \right) \right] \right\} \\ \left\{ \left[ \tau_m - \left( \frac{H_m}{c} + \delta \right) \right] - \left[ \tau_i - \left( \frac{H_i}{c} + \delta \right) \right] \right\} = 0$$

After collecting terms, the  $\delta$ 's drop out

$$c (\tau_n - \tau_j - \tau_m + \tau_i) = H_n - H_j - H_m + H_i$$

Each  $H$ , say  $H_n$ , consists of two parts: a distance  $L_n$  from the receiver to the missile, which is known to the receiver, and a distance  $r_n$  from the transmitter to the missile, which is not known at the receiver.

$$H_n = r_n + L_n$$

Using this, we can write

$$c (\tau_n - \tau_j - \tau_m + \tau_i) - L_n + L_j + L_m - L_i = r_n - r_j - r_m + r_i$$

This equation will be called the trace equation, and surfaces or curves in space that satisfy it will be called traces. In this form, the left hand side of the trace equation consists entirely of known quantities. Note that we may choose, if we wish,  $j = m$ .

Before further discussion, a coordinate system will have to be introduced. Although one might be tempted to choose spherical coordinates because this problem takes place on the earth's surface, the unknown  $r$ 's are distances, and the expression for distance is generally simpler in rectangular coordinates. Therefore, three-dimensional rectangular coordinates ( $X$ ,  $Y$ ,  $Z$ ) will be used.

If the location of the transmitter is known approximately, it is advantageous to orient the coordinate system so that the  $Z$  coordinate of the transmitter is reduced to zero or very nearly so. That is, the  $XY$  plane should cut the surface of the earth near an expected location

of the transmitter. This procedure has the advantage of reducing the problem to a two-dimensional one, which is more tractable, although this simplification is not essential.

A useful suggestion, due to Mr. J. E. Tompkins is first to calculate one or two sets of data on a three-dimensional basis, using an arbitrary orientation of coordinates. These results may then be used to reorient the coordinate system so as to reduce the problem to a two-dimensional one.

Let the receiver be at the origin (0,0,0); the unknown transmitter location will be ( $\alpha, \beta, \gamma$ ). The numbers ( $X_n, Y_n, Z_n$ ) will denote the position of the missile when it operates upon the  $n$ th pulse. With these conventions, the trace equation becomes

$$c(\tau_n - \tau_j - \tau_m + \tau_1) - L_n + L_j + L_m - L_1 = \sqrt{(X-x_n)^2 + (y-Y_n)^2 + (z-Z_n)^2} \\ - \sqrt{(x-X_j)^2 + (y-Y_j)^2 + (z-Z_j)^2} - \sqrt{(x-X_m)^2 + (y-Y_m)^2 + (z-Z_m)^2} \\ + \sqrt{(x-X_1)^2 + (y-Y_1)^2 + (z-Z_1)^2}$$

The trace surfaces (or, in degenerate cases, curves) always contain the point ( $\alpha, \beta, \gamma$ ), for any choice of  $n, j, m, i$ , subject to the condition  $n-j = m-1$ . Since the surfaces are generally not identical parallel or nested for different choices of the subscripts, then they must intersect. In the case of surfaces, at least three must be used to define a point. In case a few surfaces (or curves) have multiple intersections, then more surfaces must be drawn to eliminate the spurious intersections from the solution. Except probably in the case where the missile path is a straight line, the point ( $\alpha, \beta, \gamma$ ) will normally be the only point of intersection of all the surfaces or curves.

In brief, then, the transmitter is located by finding the simultaneous solution of a set of trace equations. If the physical situation is such that the problem can be studied well enough in two dimensions, all that is required is to graph the real solutions of each trace equation, and select the common intersection point. For a three-dimensional problem, solid or matrix graphs are still a possibility, but a better approach might be to program the set of trace equations on a digital computer.

### 3. ERROR TREATMENT

It is desirable to estimate the error that will occur when applying this method. Such error will arise from imprecisions in the measurements of times, coordinates, or distances involved. Let  $\sigma$  be any one of the

numerical parameters involved in a given trace equation, and suppose that an estimate of the error of measurement of  $\sigma$  shows that  $\sigma$  lies within the bounds  $K \leq \sigma \leq \lambda$ . The symbol  $\bar{\sigma}$  will denote the mean value (in any sense) of these measurements of the parameter.

In this procedure, the graphical presentation is essential. First, graph, for each trace equation, the surface or curve using the value  $\bar{\sigma}$ ; this will be called the median surface (curve). Now as  $\sigma$  varies from  $\bar{\sigma}$ , the trace surface deforms continuously in at least the 6-space in which  $x$ ,  $y$ , and  $z$  are complex quantities. The new surfaces created by varying  $\sigma$  from  $\bar{\sigma}$  will be called the varied surfaces (curves). This smooth variation is due to the fact that the trace equations are continuous functions of all variables and parameters involved. For values of  $(x, y, z)$  in the immediate neighborhood of  $(\alpha, \beta, \gamma)$ , the solution of each trace equation will be wholly real.

Consider any point  $P$  on the median surface (curve); the varied surfaces (curves) will generally lie on both sides of the median curve, depending on which way  $\sigma$  varies from  $\bar{\sigma}$ . Denote the distance at  $P$  to a varied curve by

$$[P - p]$$

and arbitrarily denote the distance on one side of the median curve +, on the other side -. The point  $p$  will be the point on the varied surface closest to  $P$ .

For each  $P$ , there are points  $q, p, r$  on the varied surfaces, such that

$$[P - q] = \sup [P - p]$$

$$[P - w] = \inf [P - p]$$

where  $w$  and  $q$  are found by varying  $\sigma$  within the prescribed limits. A surface is then defined by the  $q$ 's and another by the  $w$ 's; these surfaces (or curves) are called extreme surfaces. The volume (or area) they enclose will be called the error volume (area)  $E_k$ ; the subscript denotes that it was generated from the  $k$ th trace equation.

A little thought will now show that the target will lie in the volume (area)  $A$  given by

$$A = E_1 \cap E_2 \cap \dots \cap E_w$$

if there were  $w$  trace equations employed altogether.

If, instead of only one parameter, several parameters have error tolerances, the principle remains the same. The parameters are varied, singly or in any combination, within the limits of their respective errors, until a  $q$  and  $w$  are defined for each  $P$ , and the extreme surfaces constructed as before.

It will be recognized that the extreme surfaces do not necessarily coincide with any of the varied surfaces. The extreme surfaces are rather to be considered as envelopes; as such, they must satisfy the differential equation of a family of curves, at least as a singular solution. The envelope need not be a member of the family itself.

In concluding the general discussion of the problem, attention should be called to the assumption that the pulse repetition frequency (prf) is constant. Although this point has not been rigidly investigated, it seems reasonable that this requirement can be relaxed somewhat. For example, if the prf jitters around an average value, this will seemingly not lead to any great error over a large number of observations. It is also very likely that, if data be examined over any long time interval during which the prf has a time average, useful results will be obtained, even if the momentary excursions of the prf are large. Even if some error is caused by nonconstancy of the prf, in some applications even imprecise results are better than no information at all.

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